Assignment 8

True/False questions, not to turn in:

- $\phi(n)$ is always strictly less than n, when $n \geq 2$.
- If a, b are reduced residues modulo n, then so is $a+b$.
- If a, b are reduced residues modulo n, then so is ab.
- If a is a reduced residue modulo n, and $m|n$, then a is also a reduced residue modulo m.
- The multiplicative inverse of an element a modulo a prime p is a power of the element.
- $\phi(n)$ is always an even number.
- $\phi(n)$ is always divisible by 4.
- There are $n > 2$ and a such that the multiplicative inverse of a modulo n is equal to $-a (= n - a)$.

Questions to turn in:

1. Using the formulas we have learned for ϕ, compute $\phi(215)$. Also determine at least 5 elements that are reduced residues modulo 215.
2. Using Euler's theorem followed by fast exponentiation, compute $11^{214563} \bmod 215$. Only use your calculator for computing the product of two numbers and for reducing numbers modulo another (i.e. don't just give something like $11ˆ{313} to the calculator — exponent made up for illustrative purposes).
3. Use some of the work from problem 2 to find the multiplicative inverse of 11 modulo 215.
4. Find all reduced residues in $\mathbb{Z}_{16}$, and for each residue find the first power that equals 1. Determine if there is a reduced residue, such that looking at its powers produces all the other reduced residues.
5. Do the same for $\mathbb{Z}_{18}$.