Assignment 1

Make sure to write complete proofs. Try to avoid skipping steps. Write clear sentences.

1. A "multiplicative inverse" for a number x is a number y such that $xy = 1$.

   i. Show that every non-zero rational number has a multiplicative inverse (that is also a rational number). Don't just write $1/x$, the point of this question is to show that "$1/x$" exists. Assume that the rational number is written in the form that rational numbers are, and use that to explicitly write down the inverse.

   ii. If z is a complex number $a + bi$, then we define the *conjugate* $\bar{z}$ as $\bar{z} = a - bi$. Show that the product of a complex number with its conjugate is a real number.

   iii. Show that every non-zero complex number has a multiplicative inverse.

2. For this question assume the following for integers, which is the analog of "every integer is odd or even" but using 3 instead of 2 as a factor. We have not proven this assumption, but you may, and will need to, use it. The assumption is: Every integer n can be written in exactly one of the following 3 ways:

   Type A In the form $3k$ where k is some integer.
   Type B In the form $3k + 1$ where k is some integer.
   Type C In the form $3k + 2$ where k is some integer.

   Answer the following questions:

i. Show that if n is of type C, then $2n$ is of type B.

ii. Show that if m is of type A and n is any integer, then mn is also of type A.

iii. Show that if n is an integer, then n^2 cannot be of type C.

iv. Show that for any integer n, the product $(2n + 1)(n + 1)n$ is of type A.